Introduction to Integration

Math 130 - Essentials of Calculus

22 November 2019

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$$\begin{aligned}
F(x) &= x^2 + 5 \\
F(x) &= x^2 - \pi \\
F(x) &= x^2 - 2 \\
F(x) &= x^2 + 10000 \\
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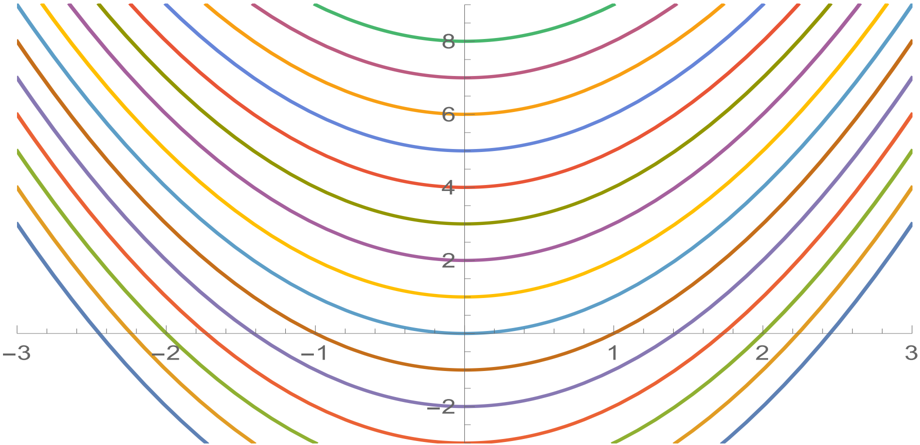
How do all of these relate to each other?

How Antiderivatives Differ

Here is a plot of various antiderivatives of $f(x) = 2x$.



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Note that we can use **any** antiderivative $F(x)$, though we usually take the simplest one in practice.

Examples

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Find the general antiderivatives of the following functions

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6. $f(x) = 2x + 4$

Notation for Antiderivatives

As it is awkward to keep saying "the general antiderivative of $f(x)$ is $F(x) + C$," we use the notation of an **indefinite integral.** That is,

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- $\int kf(x)\,dx = k \int f(x)\,dx$

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5. $\int (x^3 - 3x + 1) dx$
6. $\int 3^x dx$